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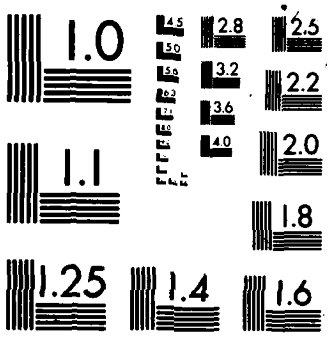

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by

Kao Zhi and Lu Wen-Qiang

FOREIGN TECHNOLOGY DIVISION

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ON WAVES IN NONADIABATIC AND NONEQUILIBRIUM GASES

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Abstract

This paper deals with the propagation of the waves (including relaxation, pressure, and density waves) and thermal mode in nonadiabatic and nonequilibrium gases. Based on the mechanism of thermal disturbance feedback, a new governing equation for the propagation of disturbances and the dispersion relation has been obtained. Some new conclusions are reached. For example, the disturbance is either purely growing or slow damped which presents a reasonable explanation for abnormal absorptive phenomena of infrasonic sound waves in atmosphere[9]; nonadiabatic characteristic of gas can cause a remarkable dispersion phenomena; the propagation of noise can be damped, etc. In addition, the conclusion that pressure and density waves and thermal mode may be unstable agrees with the experimental fact about thermal instability in high power laser discharges[6,7].

I. INTRODUCTION

Much research has been carried out on the absorption and dispersion of sound waves in adiabatic and nonequilibrium gases and liquids [1]-[3]. Analyses of the effect of nonadiabatic effects on the propagation of disturbances, i.e., thermal disturbances, have produced published works that agree with the analyses using gasdynamic acoustics. Thermal disturbances are usually treated like acoustic sources [4], [5], and the wave operators used are still the canonical acoustic wave operators. However, under many practical conditions, such as thermal disturbances in gaseous discharges, radiative heating and turbulent flow heat exchanges, the thermal disturbances are not only dependent on the state parameters of the gas but are also affected by the perturbations in these state parameters. For example, in gaseous discharges, the thermal disturbance is dependent on the electrical conductivity of the gas [6], [7]. It is, therefore, both a function of the density of the gas and a function of plasma heating induced by lasers [8]; the thermal disturbance is dependent on the electrical conductivity of the plasma, which is a function of

the density and temperature of the plasma as well as the perturbations in these quantities. In other words, perturbations in the state parameters (such as pressure, density, etc.) give rise to nonadiabatic thermal disturbances. These thermal disturbances will have positive or negative feedback effects, and the governing equation and wave operator for thermal disturbances will, as a result, be different from the canonical equations and operators. Based on this fundamental concept, we have derived a governing equation for the propagation of disturbances in nonadiabatic and nonequilibrium gases. Using this new equation as a starting point, we have analyzed dispersion relations, damping and amplification characteristics, discussed the propagation of the relaxation, density and pressure waves and thermal wave modes in nonadiabatic and nonequilibrium gases, and analyzed the interactions among these wave modes.

Some new results have been obtained in this study that differ from those obtained by treating thermal disturbances as acoustic sources [4], [5]. The fundamental reason for the difference is: thermal disturbances treated as acoustic sources are independent of the perturbations in the state parameters of the gas (such as the perturbation p' in pressure, the perturbation ρ' in density, etc.), but thermal disturbances with feedback mechanism are dependent on the perturbations in the state parameters. Therefore, by analyzing the mechanism of thermal disturbance feedback, we may obtain a better understanding of the nonadiabatic process and the interaction among the different wave modes.

II. FUNDAMENTAL EQUATION

We will not limit ourselves to any specific nonequilibrium process, but will, like general treatments of the subject [1], [3], consider a general nonequilibrium process. This could be the excitation and relaxation of molecular vibrations, the excitation and relaxation of the rotation of molecules with high rotational inertia, or some chemical reaction. Individual nonequilibrium processes may be represented by means of a nonequilibrium variable q and a relaxation

characteristic time τ. The mass conservation, momentum conservation, energy conservation equations, nonequilibrium rate equation and state equation for a nonadiabatic and nonequilibrium gas are, respectively,

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \frac{\partial u_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{Du_i}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial x_i} = 0, \tag{2}$$

$$\rho\frac{Dh}{Dt} - \frac{Dp}{Dt} = \rho\bar{q}, \tag{3}$$

$$\frac{Dq}{Dt} = \frac{q^* - q}{\tau}, \tag{4}$$

$$h = h(p, \rho, q), \tag{5}$$

in the above $\frac{D}{Dt} = \frac{\partial}{\partial t} + u_i\frac{\partial}{\partial x_i}$, p, ρ and h are, respectively, pressure density and enthalpy. u_i is the velocity component, q^* is the equilibrium value of the nonequilibrium quantity q, τ is the relaxation characteristic time of the nonequilibrium process denoted by q, and $\bar{q}$ is the nonadiabatic heat source term. For the unperturbed gas, we have

$$p = p_0 + p', \ \rho = \rho_0 + \rho', \ u_i = u_i',$$
$$h = h_0 + h', \ q = q_0 + q', \ q^* = q_0^* + q^{*\prime}. \tag{6}$$

Here, the subscript 0 denotes the unperturbed state, and the (') denotes perturbation. As stated above, the thermal perturbation $\bar{q}'$ of the nonadiabatic term $\bar{q}$ should be assumed to be a function of the perturbations, p', ρ' and q'. For instance, in the case of gaseous discharges, a perturbation in density produces changes in electrical conductivity, which in turn cause variations in local heating of the gas, giving rise to nonadiabatic thermal disturbances [6]. Hence, in general, nonadiabatic thermal disturbances may be expanded as follows:

$$\bar{q}' = \bar{q} - \bar{q}_0 = \bar{q}_p p' + \bar{q}_\rho \rho' + \bar{q}_q q'. \tag{7}$$

Here, $\bar{q}_p = \left(\frac{\partial \bar{q}}{\partial p}\right)_0$, and the other terms are similarly defined.

Substituting equations (6) and (7) into equations (1)-(4), and making use of equation (5), we obtain, after carrying out appropriate

operations and simplifications,

$$\tau_0^+ \frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}\Box_f \rho' + Q'_p \frac{\partial^2 \rho'}{\partial t^2} + Q'_\rho \frac{\partial^2 \rho'}{\partial x_i^2}\right] + \frac{\partial}{\partial t}\Box_e \rho' + \frac{\tau_0^+}{\tau_0}\left(Q'_p \frac{\partial^2 \rho'}{\partial t^2} + Q'_\rho \frac{\partial^2 \rho'}{\partial x_i^2}\right) = 0, \tag{8}$$

where

$$\Box_f = \frac{1}{a_f^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_i^2}, \tag{9}$$

$$\Box_e = \frac{1}{a_e^2}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial x_i^2}, \tag{10}$$

$$a_f^2 = -\rho_0 h_\rho (\rho_0 h_p - 1)^{-1}, \tag{11}$$ 715

$$a_e^2 = -\rho_0 (h_\rho + h_q q_\rho^*)(\rho_0 h_p - 1 + \rho_0 h_q q_p^*)^{-1}, \tag{12}$$

$$Q'_p = \frac{1}{h_\rho}(\bar{q}_p + \bar{q}_q q_p^*), \tag{13}$$

$$Q'_\rho = \frac{1}{h_\rho}(\bar{q}_\rho + \bar{q}_q q_\rho^*), \tag{14}$$

$$\frac{\tau_0}{\tau_0^+} = \frac{1}{h_\rho}(h_\rho + h_q q_\rho^*). \tag{15}$$

Here $h_\rho = \left(\frac{\partial h}{\partial \rho}\right)_0$, , and the other terms are similarly defined. Equation (8) is the governing equation for the propagation of disturbances in a nonadiabatic and nonequilibrium gas.* $\Box_e$ is the canonical frozen wave operator expressed in terms of a_c. τ_o^+ is on the same order of magnitude as τ_o. It denotes the characteristic relaxation time of the interaction of the nonequilibrium process and the wave motion. $\tau_o^+ \to 0$ denotes the condition where the waves are propagated in a gas in equilibrium, which we refer to as the equilibrium gas condition. $\tau_o^+ \to \infty$ denotes the frozen gas condition. It can be easily shown that a_f and a_c are the frozen wave sound velocity and equilibrium sound velocity [2], [3] in the ordinary sense. In equation (8), the four terms besides the canonical frozen wave operator and the canonical equilibrium wave operator are correction terms for the wave operator due to the nonadiabatic thermal disturbances. Under adiabatic conditions, i.e., $Q'_p = Q'_\rho = 0$, equation (8) reduces to the ordinary nonequilibrium acoustic equation [2], [3]. When $\tau_o^+ \to \infty$ and 0, equation (8) reduces, respectively, to the governing

*$\Box_f$ is the canonical frozen wave operator expressed in terms of a_f while

equation for the propagation of disturbances in a nonadiabatic frozen gas and that in a nonadiabatic equilibrium gas.

III. DISPERSION RELATIONS (ONE-DIMENSIONAL VALVE)

It will not be easy to obtain from equation (8) a general solution that is as simple in form as that for the canonical acoustic equation. For convenience, we first consider the propagation of a small amplitude simple harmonic motion along the x-direction. Such a disturbance can be produced by a valve undergoing small amplitude simple harmonic motion at x = 0 (see Figure 1).

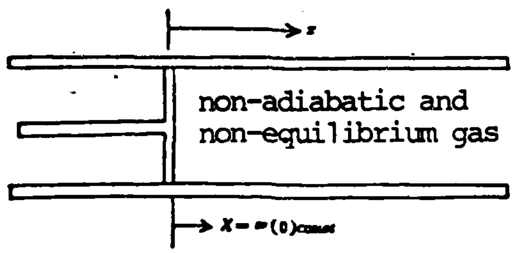


Figure 1. Valve undergoing simple harmonic motion.

Making use of the following one-dimensional flow relation

$$\rho_0 \frac{\partial u'}{\partial x} = -\frac{\partial \rho'}{\partial t}.$$

we can simplify equation (8) to read

$$\tau_1^+ \frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}\left(\frac{1}{a_f^2}\frac{\partial^2 u'}{\partial t^2} - \frac{\partial^2 u'}{\partial x^2}\right) + Q'_p \frac{\partial^2 u'}{\partial t^2} + Q'_\rho \frac{\partial^2 u'}{\partial x^2}\right] + \frac{\partial}{\partial t}\left(\frac{1}{a_e^2}\frac{\partial^2 u'}{\partial t^2} - \frac{\partial^2 u'}{\partial x^2}\right) + \frac{\tau_1^+}{\tau_0}\left(Q'_p \frac{\partial^2 u'}{\partial t^2} + Q'_\rho \frac{\partial^2 u'}{\partial x^2}\right) = 0. \tag{16}$$

For small oscillations, the fluid at the surface of the valve should move at the velocity of the valve. Therefore, the boundary condition at x = 0 for the velocity of the perturbation is

$$u'(0, t) = W(0)\cos \omega t. \tag{17}$$

Equations (16) and (17) have solutions of the form $\mathrm{Re}[W(x)e^{i\omega t}]$. Substituting equation (17) into equation (16), we obtain

$$u'(x, t) = W(0)\exp\left(-\frac{\omega}{a_f}\beta x \sin\varphi\right)\cos\left(\omega t - \frac{\omega}{a_f}\beta x \cos\varphi\right), \tag{18}$$

where

$$\varphi = \frac{1}{2}\,\mathrm{tg}^{-1}\frac{\alpha_1\alpha_4 - \alpha_2\alpha_3}{\alpha_1\alpha_3 + \alpha_2\alpha_4},$$

$$\beta = \left[\frac{(\alpha_1\alpha_3 + \alpha_2\alpha_4)^2 + (\alpha_2\alpha_3 - \alpha_1\alpha_4)^2}{(\alpha_3^2 + \alpha_4^2)^2}\right]^{\frac{1}{4}}, \tag{19}$$

$$\alpha_1 = (\omega\tau_0^+)^2 - a_f^2 Q_p' \tau_0^+ \frac{\tau_0^+}{\tau_0},$$

$$\alpha_2 = -(\omega\tau_0^+)\left(\frac{a_f^2}{a_e^2} + a_f^2 Q_p' \tau_0^+\right),$$

$$\alpha_3 = (\omega\tau_0^+)^2 + Q_\rho' \tau_0^+ \frac{\tau_0^+}{\tau_0},$$

$$\alpha_4 = -(\omega\tau_0^+)(1 - Q_\rho' \tau_0^+). \tag{20}$$

It can be seen that the interaction between the wave and the non-adiabatic and non-equilibrium process depends on the ratios of the characteristic times of the four corresponding processes, viz., the three Damkohler numbers: $\omega\tau_o^+$ (ratio of the characteristic time of the relaxation process to the characteristic time ω^{-1} of the oscillation of the valve), $a_f^2 Q_p' \tau_0^+$ (ratio of the relaxation characteristic time to the characteristic time $(a_f^2 Q_p')^{-1}$ of the pressure feedback process) and $Q_\rho' \tau_0^+$ (ratio of the relaxation characteristic time and the characteristic time $(Q_\rho')^{-1}$ of the density feedback process). Figures 2-6 show the variation with $\omega\tau_o^+$ of the phase velocity $a = a_f\ (\beta \cos\phi)^{-1}$ and the damping or amplification length $L = a_f(\omega\beta\sin\varphi)^{-1}$, where $a_f^2 Q_p' \tau_0^+$, $Q_\rho' \tau_0^+$, a_f^2/a_e^2 and τ_0^+/τ_0 are used as parameters. Our analysis gives the following conclusions.

1) The phase velocity is dependent on the frequency ω, i.e., there is dispersion. Under nonadiabatic feedback conditions, there is a large variation of a with $\omega\tau_o^+$. For gases in the low frequency limit or under equilibrium conditions, i.e., when $\omega\tau_0^+ \to 0$ $a \to [|Q_\rho'|/|Q_p'|]^{1/2}$. Therefore, when $Q_p' = 0$ and $Q_\rho' \neq 0$, $a \to \infty$, while when $Q_p' \neq 0$ and $Q_\rho' = 0$, $a \to 0$. For gases in the high frequency limit or frozen gases, i.e., when $\omega\tau_o^+ \to \infty$, a is always a_f. For the variation of a with $\omega\tau_o^+$, see Figures 2 and 3.

It can be seen that the pressure feedback causes a decrease in the velocity of the low frequency waves, while the density feedback

accelerates the low frequency waves. The phase velocity of the high frequency wave is, however, not affected by nonadiabatic feedback processes. Under adiabatic conditions, i.e., when $Q'_p = Q'_\rho = 0$ a increases from its equilibrium sound velocity a_c for $\omega\tau_o^+ \to 0$ monotonically to the frozen wave sound velocity a_f for $\omega\tau_o^+ \to \infty$. For most gases, a_f is slightly higher than a_c [1]-[3]. In addition, at the low frequency limit of nonadiabatic equilibrium or frozen gases, i.e., when $\omega \to 0$, a is still equal to $[|Q'_\rho|/|Q'_p|]^{1/2}$; at the high frequency limit, i.e., when $\omega \to \infty$, a is equal to a_c (equilibrium condition) or a_f (frozen wave condition).

2) For the damping and amplification of the disturbances, the criteria for stability of disturbance are obtained from solving equations (18) and (19):

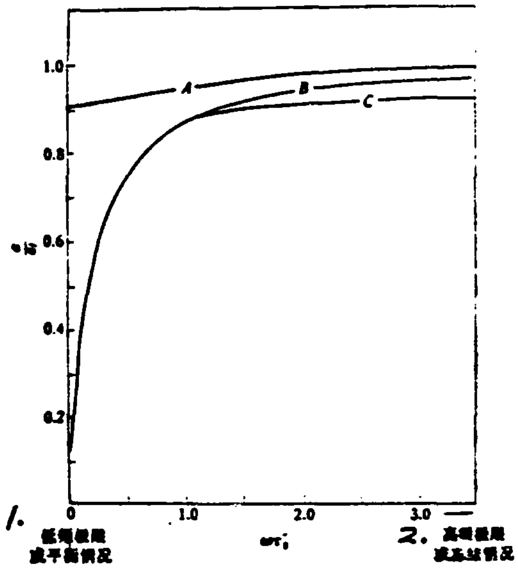
$$S = (\alpha_1\alpha_4 - \alpha_2\alpha_3)(\alpha_1\alpha_3 + \alpha_2\alpha_4) \begin{cases} > 0 & \text{stable} \\ < 0 & \text{unstable} \end{cases} \tag{21}$$

Under adiabatic conditions, $S = (\omega\tau_0^+)^5[(a_f^2/a_c^2) - 1][1 + (\omega\tau_0^+)^2]$. As $a_f > a_c$, it is always stable. Under nonadiabatic conditions, the criteria for stability are listed in Table 1.

The experimental results of thermal instability in laser discharges can be fairly well accounted for using the criteria given in Table 1. The conclusions made by Nighan, et al., [6], [7] about the thermal instability in laser discharges are equivalent to the results in Table 1 corresponding to $\tau_o^+ \to 0$ and $Q'_p = 0$. The results given in Table 1 and the present analysis further show that under the nonadiabatic condition where Q'_ρ and Q'_p are simultaneously nonzero, the region of stability is dependent on ω. When the high frequency disturbance is damped, the low frequency disturbance may be amplified, and vice versa.

3) The amplitude varies in the following manner. The amplitude of the high frequency waves (when $\omega \to \infty$ and the other parameters remain unchanged) is damped or amplified exponentially according to

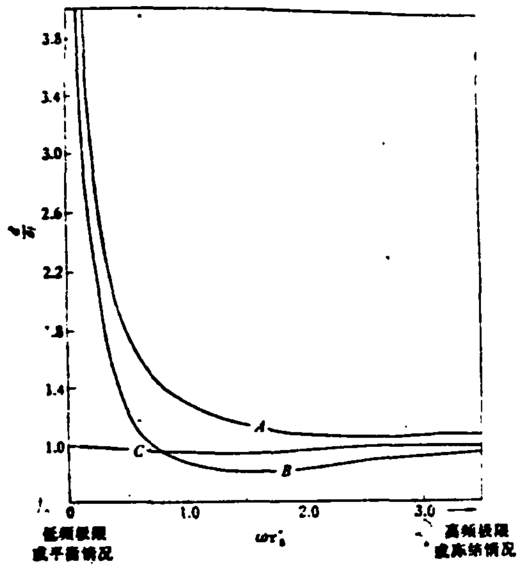
$$\exp\left[-\frac{(a_f^2/a_c^2 - 1) + (a_c^2 Q'_p + Q'_\rho)\tau_0^+}{2a_f\tau_0^+}\, x\right]. \tag{22}$$



$(a_f/a_e = 1.1,\ \tau_e^+/\tau_e = 1.1;$ 3. A指绝热情况；B和C指非绝热情况：$B: Q'_e = 0,\ a_f^2 Q'_p \tau_e^+ = 1;\ C: Q'_e = 0,\ a_f^2 Q'_p \tau_e^+ = -1)$

Figure 2. Variation of phase velocity a/a_f with $\omega\tau_o^+$.

1--low frequency limit or equilibrium condition; 2--high frequency limit or frozen wave condition; 2--A refers to adiabatic condition; B and C refer to non-adiabatic condition



$(a_f/a_e = 1.1,\ \tau_e^+/\tau_e = 1.1;\ A: Q'_p = 0,\ Q'_e \tau_e^+ = 1;\ B: Q'_p = 0,\ Q'_e \tau_e^+ = -1;\ C: a_f^2 Q'_p \tau_e^+ = 1,\ Q'_e \tau_e^+ = -1)$

Figure 3. Variation of phase velocity a/a_f with $\omega\tau_o^+$.

1--low frequency limit or equilibrium condition; 2--high frequency limit or frozen wave condition

TABLE 1

	$\omega\tau_e^+ \to 0$	$\omega\tau_e^+ \to \infty$	$\tau_e^+ \to 0;\ \infty$
stable	$Q'_e Q'_p > 0 \quad Q'_p$ (或 Q'_e) < 0 $Q'_e Q'_p = 0 \quad Q'_p\left[\frac{a_f^2}{a_e^2} + a_f^2 Q'_p \tau_e^+\left(1 - \frac{\tau_e^+}{\tau_e}\right)\right] > 0$ $Q'_e\left[\frac{a_f^2}{a_e^2} + Q'_e \tau_e^+\left(\frac{\tau_e^+}{\tau_e} - 1\right)\right] > 0$ $Q'_e Q'_p < 0 \quad a_e^2 Q'_p + Q'_e > 0$	$(Q'_e + a_f^2 Q'_p)\tau_e^+$ $> \left(1 - \frac{a_f^2}{a_e^2}\right)$	$\tau_e^+ \to 0$ $(a_e^2 Q'_p + Q'_e)\left[\omega^2 - a_e^2 Q'_p Q'_e \left(\frac{\tau_e^+}{\tau_e}\right)^2\right]$ > 0 $\tau_e^+ \to \infty$ $(a_f^2 Q'_p + Q'_e)(\omega^2 - a_f^2 Q'_p Q'_e) > 0$
unstable	$Q'_e Q'_p > 0 \quad Q'_p$ (或 Q'_e) > 0 $Q'_e Q'_p = 0 \quad Q'_p\left[\frac{a_f^2}{a_e^2} + a_f^2 Q'_p \tau_e^+\left(1 - \frac{\tau_e^+}{\tau_e}\right)\right] < 0$ $Q'_e\left[\frac{a_f^2}{a_e^2} + Q'_e \tau_e^+\left(\frac{\tau_e^+}{\tau_e} - 1\right)\right] < 0$ $Q'_e Q'_p < 0 \quad a_e^2 Q'_p + Q'_e < 0$	$(Q'_e + a_f^2 Q'_p)\tau_e^+$ $< \left(1 - \frac{a_f^2}{a_e^2}\right)$	$\tau_e^+ \to 0$ $(a_e^2 Q'_p + Q'_e)\left[\omega^2 - a_e^2 Q'_p Q'_e \left(\frac{\tau_e^+}{\tau_e}\right)^2\right]$ < 0 $\tau_e^+ \to \infty$ $(a_f^2 Q'_p + Q'_e)(\omega^2 - a_f^2 Q'_p Q'_e) < 0$

It is clear that under adiabatic conditions, the amplitude is damped exponentially. Under nonadiabatic and frozen way conditions, the amplitude of the high frequency waves is damped or amplified, respectively, as

$$\exp\left(-\frac{a_f^2 Q'_\rho + Q'_\rho}{2a_f}\frac{\tau_0^+}{\tau_0}x\right),$$
$$\exp\left(-\frac{a_f^2 Q'_p + Q'_\rho}{2a_f}x\right) \quad . \tag{23}$$ 718

The amplitude of the low frequency waves (when $\omega \to \infty$ and the other parameters remain unchanged) is damped or amplified exponentially as

$$\exp\left\{\pm\frac{\omega^{3/2}}{a_e}\left[(a_f^2 Q'_p + Q'_\rho)\frac{\tau_0}{\tau_0^+}\right]^{1/2}\frac{x}{Q'_\rho}\right\} \quad . \tag{24}$$

In the special case where $Q'_p = 0$ or $Q'_\rho = 0$, the amplitude of the low frequency waves is damped or amplified, respectively, according to

$$\exp\left[\pm\frac{\omega^{3/2}}{a_e}\left(\frac{\tau_0}{\tau_0^+|Q'_\rho|}\right)^{1/2}x\right] \quad (Q'_p = 0),$$
$$\exp\left[\pm\omega^{1/2}\left(|Q'_p|\frac{\tau_0^+}{\tau_0}\right)^{1/2}x\right] \quad (Q'_\rho = 0) \; . \tag{25}$$

This is obviously very different from the exponential damping [3] of the amplitude of the low frequency waves under the adiabatic condition $Q'_p = Q'_\rho = 0$, according to

$$\exp\left[-\frac{\omega^2}{2a_e}\left(1-\frac{a_e^2}{a_f^2}\right)\tau_0^+ x\right] \cdot$$

4) Damping, amplification and slow damping: It can be seen from equations (18), (19) and (20) that when the feedback parameters Q'_p and Q'_ρ are appropriately matched, the nonadiabatic and nonequilibrium characteristics of the gas cause amplification of the perturbation at certain frequencies, and damping or slow damping of the perturbation at other frequencies. Figure 4 gives an example of such a complicated situation. Within the range of $0 < (\omega\tau_0^+)^2 < 1.1$, the perturbation is amplified; in the region where $(\omega\tau_0^+)^2 > 1.1$, the perturbation is damped; in the neighborhood of $(\omega\tau_0^+)^2 \approx 1.1$, the damping length and amplification length both tend to infinity, forming, in

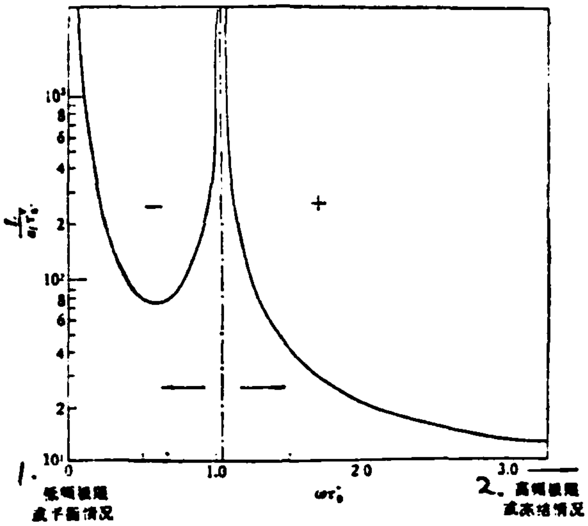
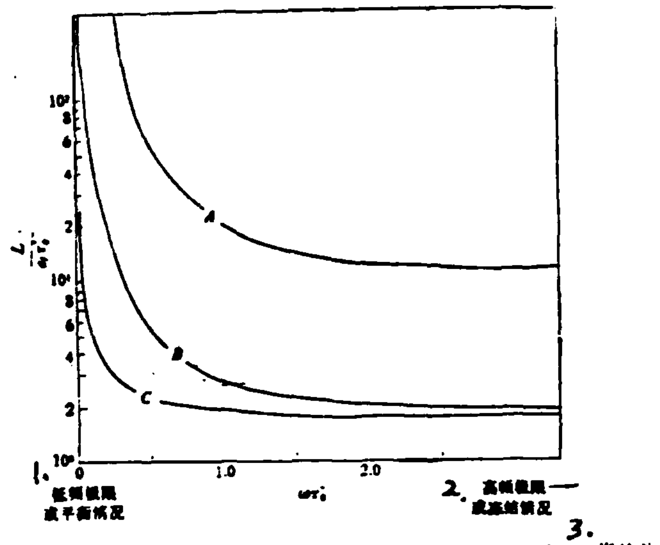


Figure 4. Variation with $\omega\tau_0^+$ of the amplification-damping length $L/a_f\tau_0^+$ ($a_f/a_0 = 1.1$, $\tau_0^+/\tau_0 = 1.1$, $a_f' Q_p' \tau_0^+ = 1$, $Q_p' \tau_0^+ = -1$;
+ denotes the region of damping, where L > 0; - denotes the region of amplification where L < 0; -•- is where $(\omega\tau_0^+)^2 = 1.1$)

1--low frequency limit or equilibrium condition;
2--high frequency limit or frozen wave condition

effect, a region of slow damping. This effect of nonadiabatic feedback can be used to fairly satisfactorily account for the abnormal absorptive phenomena of sonic and infrasonic waves in the atmosphere [9]. Hence, nonadiabatic feedback provides a reasonable mechanism for explaining these abnormal absorptive phenomena [9]. Figure 5 gives an example in which the perturbation is damped in all frequency ranges. When the case of nonadiabatic feedback is compared with the adiabatic case, the former has a damping length $10\text{-}10^2$ times shorter than that of the latter. Figure 6 gives an exmaple 719
in which the perturbation is amplified in all frequency ranges.

5) Damping effect of nonadiabatic feedback on the propagation of noise: The process of nonequilibrium relaxation is a type of nonuniformity in the gaseous medium, and like other types of nonuniformity, can have a damping effect on the propagation of gas-dynamic noises [5]. This is in agreement with the conclusion given in [5]. It has been further found out here that the nonequilibrium process has a relatively large damping effect on the high-frequency



3.
衰减长度 $L/a_f\tau_0^+$ 随 $\omega\tau_0^+$ 的变化($a_f/a_e=1.1$, $\tau_0^+/\tau_e=1.1$; A:指绝热情况; B和C指非绝热情况: B:$Q'_p=0$, $Q'_p\tau_0^+=1$; C:$Q'_e=0$, $a_f Q'_e\tau_0^+=1$)
4.

Figure 5. Variation with $\omega\tau_o^+$ of the damping length $L/a_f\tau_o^+$
1--low frequency limit or equilibrium condition; 2--high frequency limit or frozen wave condition; 3--A refers to adiabatic condition; 4--B and C refer to nonadiabatic condition

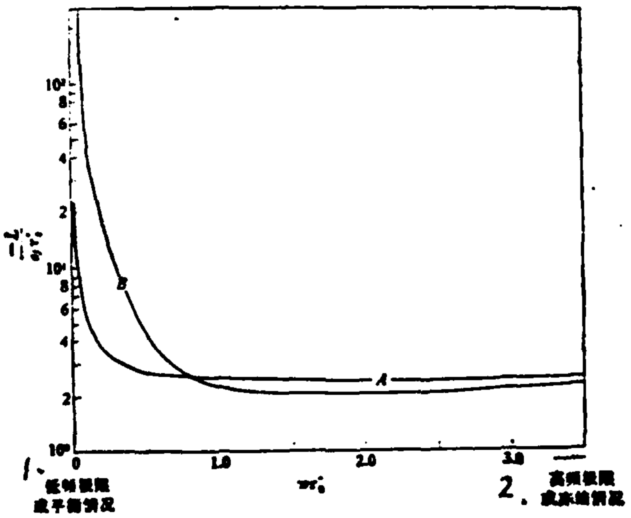


Figure 6. Variation with $\omega\tau_o^+$ of the amplification length $-L/a_f\tau_o^+$
1--low frequency limit or equilibrium condition;
2--high frequency limit or frozen wave condition

waves, but its damping effect on the low frequency waves is rather small. The nonadiabatic process produces effective damping. For instance, if the conditions $Q'_pQ'_\rho \leqslant 0$, and $a_e^2Q'_p + Q'_\rho > 0$ are met simultaneously, then effective damping can be obtained in all frequency ranges for a gas in equilibrium. Please refer to curve C in Figure 5.

IV. RELAXATION, PRESSURE AND DENSITY WAVES AND THERMAL MODES

To further illustrate the effect of the nonadiabatic process on the propagation of perturbation, we carry out a Fourier transformation of equation (8) with respect to the space variables, and obtain

$$\tau_0^+\left[\frac{1}{a_f^2}\frac{d^4F}{dt^4} + k^2\frac{d^2F}{dt^2} + Q'_p\frac{d^3F}{dt^3} - k^2Q'_\rho\frac{dF}{dt}\right]$$ 720

$$+\frac{1}{a_e^2}\frac{d^3F}{dt^3} + k^2\frac{dF}{dt} + \frac{\tau_0^+}{\tau_0}\left(Q'_p\frac{d^2F}{dt^2} - k^2Q'_\rho\frac{dF}{dt}\right) = 0. \qquad (26)$$

In the above, F is the Fourier transform of $\rho'(r, t)$

$$F = \frac{1}{(2\pi)^3}\int \rho'(r, t)e^{-ik\cdot r}dr, \qquad (27)$$

$$\rho'(r, t) = \int F(k)e^{ik\cdot r}dk \qquad (28)$$

Equation (27) is integrated over all physical space, while equation (28) is integrated over the entire three-dimensional wave number space (k_1, k_2, k_3), $k = |k|$. It is not very hard to obtain the solution of equation (26) in exponential form, but the process is tedious. In what follows we give, through an analysis of certain limiting conditions, the relaxation, pressure and density waves and the thermal modes, and discuss their main characteristics, interrelationship and stability.

1) Thermal feedback due to pressure perturbation under equilibrium conditions ($\tau_0^+ \to 0$, $Q'_\rho = 0$) . The complex solution for the wave component corresponding to wave number k, as obtained from equation (26), is

$$\rho'_k(r, t) \sim A_1\exp\left\{ik\cdot r + \frac{t}{2}\left[-Q'_p\frac{\tau_0^+}{\tau_0} + \left(\left(Q'_p\frac{\tau_0^+}{\tau_0}\right)^2 - 4k^2a_e^2\right)^{1/2}\right]\right\}$$

$$+ A_2\exp\left\{ik\cdot r + \frac{t}{2}\left[-Q'_p\frac{\tau_0^+}{\tau_0} - \left(\left(Q'_p\frac{\tau_0^+}{\tau_0}\right)^2 - 4k^2a_e^2\right)^{1/2}\right]\right\}, \qquad (29)$$

where A_1 and A_2 are undetermined constants. We define the heat wave modes to be the zero-frequency modes corresponding to the condition $2ka_e \leq |Q'_p| \tau_e^+/\tau_e$ (i.e., for smaller wave numbers k), and call them thermal modes for short. We can see from equation (29) that when $Q'_p < 0$, the thermal modes grow exponentially with time; when $Q'_p > 0$, these are exponentially damped with time. For higher wave numbers, i.e., those values of k that satisfy the condition $2ka_e > |Q'_p| \tau_e^+/\tau_e$, waves that are damped with time exponentially (if $Q'_p > 0$) are produced*. Such waves that have arisen because of thermal feedback due simply to pressure perturbation are called pressure waves. Under the condition $Q'_p = 0$, i.e., adiabatic condition, equation (29) reduces to that for an undamped canonical sound wave in equilibrium, with phase velocity a_e.

2) Thermal feedback due to density perturbation under equilibrium conditions ($\tau_e^+ \to 0,\ Q'_p \to 0$) . In this case, the complex solution for the wave component corresponding to the wave number k, as obtained from equation (26), is

$$\rho'_k(\boldsymbol{r}, t) \sim \sum_{j=1}^{3} A_j \exp\Big\{ i\boldsymbol{k}\cdot\boldsymbol{r} + (ka_e)^{2/3} t \Big[\Omega_j^{-1} \Big(\frac{1}{2} Q'_\rho \frac{\tau_e^+}{\tau_e} + \Big(\Big(\frac{1}{2} Q'_\rho \frac{\tau_e^+}{\tau_e} \Big)^2 + \frac{1}{27} k^2 a_e^2 \Big)^{1/2} \Big)^{1/3} + \Omega_j^{-1} \Big(\frac{1}{2} Q'_\rho \frac{\tau_e^+}{\tau_e} - \Big(\Big(\frac{1}{2} Q'_\rho \frac{\tau_e^+}{\tau_e} \Big)^2 + \frac{1}{27} k^2 a_e^2 \Big)^{1/2} \Big)^{1/3} \Big] \Big\}, \tag{30}$$

$$\Omega_1 = \frac{-1 + i\sqrt{3}}{2}, \quad \Omega_2 = \frac{-1 - i\sqrt{3}}{2}, \tag{31}$$

where A_j (j = 1, 2, 3) are undetermined constants. The zero frequency mode of j = 1 in equation (30) is defined as thermal mode as before. The waves corresponding to j = 2,3 in equation (30) are density waves due to simple density feedback conditions. If $Q'_\rho > 0$ then the thermal mode grows with time and the density waves are damped with time; when $Q'_\rho < 0$, then the thermal mode is damped with time, and the density waves grow with time. Under adiabatic conditions, i.e., $Q'_\rho = 0$, equation (30) reduces to that for an undamped canonical sound wave in equilibrium, with phase velocity a_e. Thus, for a gas in

*or waves that grow exponentially with time (if $Q'_p < 0$)

equilibrium, thermal feedback due to density perturbation results in the simultaneous appearance of the thermal mode and the density waves. When the thermal mode is stable ($Q'_\rho < 0$), the density waves diverge; when the density waves are stable ($Q'_\rho > 0$), the thermal mode diverges. This conclusion agrees completely with that made in [7] regarding the instability in laser discharge thermal modes.

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3) Adiabatic and nonequilibrium conditions ($\tau_0^+ \neq 0$, $Q'_\rho = Q'_p = 0$). In this case, the complex solution for the wave component corresponding to k, as obtained from equation (26), is

$$\rho'_k(\boldsymbol{r}, t) \sim \sum_{j=1}^{3} A_j \exp(i\boldsymbol{k}\cdot\boldsymbol{r} + \omega_j t), \tag{32}$$

$$\begin{aligned} \omega_j \tau_0^+ = & -\frac{a_f^2}{3a_e^2} + \Omega_1^{j-1}\left\{-\frac{q^{**}}{2} + \left[\left(\frac{q^{**}}{2}\right)^2 + \left(\frac{p^*}{3}\right)^3\right]^{1/2}\right\}^{1/3} \\ & + \Omega_2^{j-1}\left\{-\frac{q^{**}}{2} - \left[\left(\frac{q^{**}}{2}\right)^2 + \left(\frac{p^*}{3}\right)^3\right]^{1/2}\right\}^{1/3}, \\ q^{**} = & (ka_f\tau_0^+)^2\left(1 - \frac{a^2}{3a_e^2}\right) + \frac{2}{27}\left(\frac{a_f}{a_e}\right)^6, \\ p^* = & (ka_f\tau_0^+)^2 - \frac{1}{3}\left(\frac{a_f}{a_e}\right)^4. \end{aligned} \tag{33}$$

In equation (32), the zero-frequency mode corresponding to j = 1 is the thermal mode with infinitely high phase velocity, and the waves corresponding to j = 2,3 can be called nonequilibrium relaxation waves, or relaxation waves in short. As $q^{**} > 0$, $[(q^{**}/2)^2 + (p^*/3)^3] \geq 0$, the thermal mode is always damped with time. As the sum of the real parts of the second and third terms of equation (33) is always less than the first term, the relaxation waves are also always damped with time. It is a well known fact that in an adiabatic and non-equilibrium gas the perturbations are always damped [2].

4) Thermal feedback due to pressure perturbation under non-equilibrium conditions ($\tau_0^+ \neq 0$, $Q'_p \neq 0$, $Q'_\rho = 0$). In this case, the complex solution for the wave components obtained from equation (26) is

$$\rho'_k(r, t) \sim \sum_{j=1}^{3} A_j \exp(i\boldsymbol{k}\cdot\boldsymbol{r} + \omega_j t), \tag{34}$$

$$\omega_j = -\frac{1}{3}\left(\frac{a_f^2}{a_e^2\tau_0^+} + a_f^2 Q'_p\right) + \Omega_1^{j-1}\left\{-\frac{q_1}{2} + \left[\left(\frac{q_1}{2}\right)^2 + \left(\frac{p_1}{2}\right)^3\right]^{1/2}\right\}^{1/3}$$
$$+ \Omega_2^{j-1}\left\{-\frac{q_1}{2} - \left[\left(\frac{q_1}{2}\right)^2 + \left(\frac{p_1}{2}\right)^3\right]^{1/2}\right\}^{1/3}. \tag{35}$$

$$q_1\tau_0^{+3} = (ka_f\tau_0^+)^2 + \frac{2}{27}\left(\frac{a_f^2}{a_e^2} + a_f^2 Q'_p\tau_0^+\right)^3$$
$$-\frac{1}{3}\left(\frac{a_f^2}{a_e^2} + a_f^2 Q'_p\tau_0^+\right)\left[(ka_f\tau_0^+)^2 + a_f^2 Q'_p\tau_0^+\frac{\tau_0^+}{\tau_0}\right],$$
$$p_1\tau_0^{+2} = (ka_f\tau_0^+)^2 + a_f^2 Q'_p\tau_0^+\frac{\tau_0^+}{\tau_0} - \frac{1}{3}\left[\left(\frac{a_f}{a_e}\right)^2 + a_f^2 Q'_p\tau_0^+\right]^2.$$

Here A_j ($j \neq 1,2,3$) are undetermined constants. For the case where $Q'_p \neq 0$, in the range of wave numbers where $k^2(a_f Q'_p)^{-2} \geq 1/3$, the zero-frequency mode of $j = 1$ in equation (34) is the thermal mode, which is approximately stable. In the range of very large wave numbers, the thermal mode is stable when $Q'_p > 0$, and unstable when $Q'_p < 0$. In the range of smaller wave numbers, where $k^2 < \frac{1}{3}(a_f Q'_p)^2$, the pressure waves are stable when $Q'_p > 0$, and unstable when $Q'_p < 0$. It should be noted that under the limiting condition of a frozen gas, i.e., when $\tau_0^+ \rightarrow \infty$, the wave corresponding to $j = 1$ in equation (34) is a pressure wave for $k^2 < \frac{1}{3}(a_f Q'_p)^2$, while under adiabatic conditions, i.e., when $Q'_p \rightarrow 0$, equation (34) reduces to equation (32), and the thermal mode of $j = 1$ becomes a relaxation wave.

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V. CONCLUSION

The governing equation for the propagation of disturbances in nonadiabatic and nonequilibrium gases, equation (8), derived in this paper on the basis of the mechanism thermal disturbance feedback, can be used to accurately describe the characteristics of the propagation of these disturbances. It can also explain the experimentally observed thermal instability in laser discharges, as well as the abnormal absorptive phenomena of infrasonic waves in the atmosphere. In contrast, the regular theory in which the thermal disturbances are treated as acoustic sources cannot reflect the nonlinear characteristics of the above mentioned wave propagation.

In nonadiabatic and nonequilibrium gases, relaxation waves, pressure waves, density waves and thermal modes can be formed. These wave modes interact with one another, and possess instability. The physical mechanism underlying the instability of these wave modes is that of nonadiabatic thermal disturbance feedback due to pressure perturbation or density perturbation.

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